Single-Spin Asymmetries and Transversity

Philip G. Ratcliffe

philip.ratcliffe@uninsubria.it

Dip.to di Scienze CC.FF.MM.

Univ. degli Studi dell'Insubria*—sede di Como

* The *Insubri* were a Celtic tribe, originally from across the Alps, who in the 5th. century B.C. settled roughly the area known today as Lombardy.

Outline

- Introduction to Single-Spin Asymmetries
- Introduction to Transversity
- SSA's in Lepton-Nucleon Scattering
- SSA's in Hadron-Hadron Scattering
- Comments and Conclusions

A large part of this talk was lifted from the Physics Reports by Barone, Drago and PGR (2002) and from a forthcoming book by Barone and PGR (2002).

Therefore, much credit is due to my two collaborators.

Preamble

Single-spin asymmetries probably represent one of the oldest forms of high-energy spin measurement: the only requirement is either beam or target polarised and in the case of Λ^0 production neither!

However, after some initial interest (due to the surprisingly large experimental magnitude), a (theoretical) dark age descended on SSA's: apparently PQCD had nothing to say, save that they ought to be zero!

In contrast, today we realise that the rich phenomenology is matched by a richness of the theoretical framework in which they can be explained . . . the subject of this talk

Preamble

One might reasonably argue that the Q^2 of all existing SSA data is too low for PQCD to be applicable.

Indeed, there are several non-PQCD models that can explain some (but never all) of the data.
Examples are:

- Andersson, Gustafson and Ingelman (1979)
- DeGrand and Miettinen (1981)
- Barni, Preparata and PGR (1992)
- Soffer and Tornqvist (1992)

. . . but I shall examine SSA's within the PQCD framework.

Preamble

Transversity too has a long history:

- The concept (though not the term) was introduced by Ralston and Soper (1979) via Drell–Yan processes
- LO anomalous dimensions were first calculated by Baldracchini *et al*. (1981) ... then promptly forgotten!
- ... re-calculated by Artru and Mekhfi (1990)
- ... also unwittingly calculated (for g_2 evolution) by:
  - Kodaira *et al*. (1979)
  - Antoniadis and Kounnas (1981)
  - Bukhvostov *et al*. (1985)
  - PGR (1986)

Introduction

Generically, SSA's reflect correlations of the form

$$\vec{S} \cdot \left(\vec{P} \wedge \vec{K} \right)$$

$\vec{S}$ is a polarisation vector
$\vec{P}$ and $\vec{K}$ are particle/jet momenta

A typical example might be

$$\begin{aligned} \vec{S} &= \text{target polarisation vector (transverse)} \\ \vec{P} &= \text{beam direction} \\ \vec{K} &= \text{final-state particle direction} \end{aligned}$$

Introduction

So, polarisations involved in SSA's will typically be transverse . . . usually . . . but see later.

Transforming the basis from transverse spin to helicity

$$|\uparrow / \downarrow\rangle = \frac{1}{\sqrt{2}} \left[\, |+\rangle \pm \mathrm{i} \, |-\rangle \right]$$

any such asymmetry takes the (schematic) form

$$\mathcal{A} \sim \frac{\langle \uparrow \mid \uparrow \rangle - \langle \downarrow \mid \downarrow \rangle}{\langle \uparrow \mid \uparrow \rangle + \langle \downarrow \mid \downarrow \rangle} \sim \frac{2 \, \mathrm{Im} \, \langle + \, | - \rangle}{\langle + \, | + \rangle + \langle - \, | - \rangle}$$

The presence of both $|+\rangle$ and $|-\rangle$ in the numerator implies the involvement of a spin-flip amplitude.

Introduction

The precise form of the numerator indicates interference between amplitudes:

- one spin-flip and one non-flip
- with a relative phase difference

Kane, Pumplin and Repko (1978) realised that in the Born approximation and massless (or high-energy) limit a gauge theory such as QCD cannot furnish either requirement:

- fermion helicity is conserved
- tree diagrams are real

"... *observation of significant polarizations in the above reactions would contradict either QCD or its applicability.*"

Introduction

Efremov and Teryaev (1985) discovered a way out . . .

Consideration of the three-parton correlators involved in, *e.g.*, g_2, leads to the following observations:

- the relevant mass scale when considering helicity flip is not the current quark mass but a hadronic mass
- the pseudo-two-loop nature of the diagrams leads to an imaginary part in a particular region of partonic phase space

. . . but it was still some time before progress was made and the complexity of the available structures was really exploited—see Qiu and Sterman (1991, 1992).

Introduction

Transversity is the third (and final) twist-two partonic distribution function.
It is important to make the distinction between

- partonic distributions — $q(x)$, $\Delta q(x)$, $\Delta_T q(x)$, $\ldots$
- DIS structure functions — F_1, F_2, g_1, g_2, $\ldots$

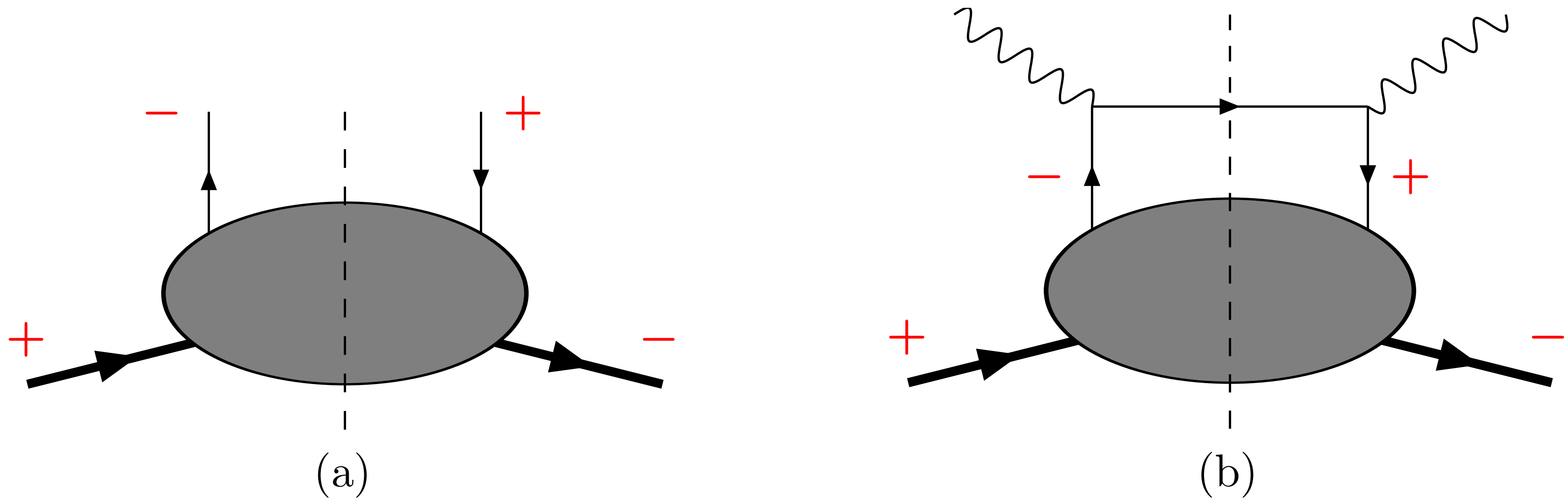
In the unpolarised and helicity-dependent cases at leading twist there is a simple correspondence between the two:

> DIS structure functions are just weighted sums of partonic distributions (or densities).

In the transverse spin case:

- there is no DIS transversity structure function
- g_2 does not correspond to any partonic density

Chirality Flip



(a) Chirally-odd hadron–quark amplitude for $\boldsymbol{h_1}$

(b) Chirality-flip forbidden DIS handbag diagram

N.B. Chirality flip is not a problem if the quarks connect to different hadrons, as in Drell–Yan.

Twist Basics and Operators

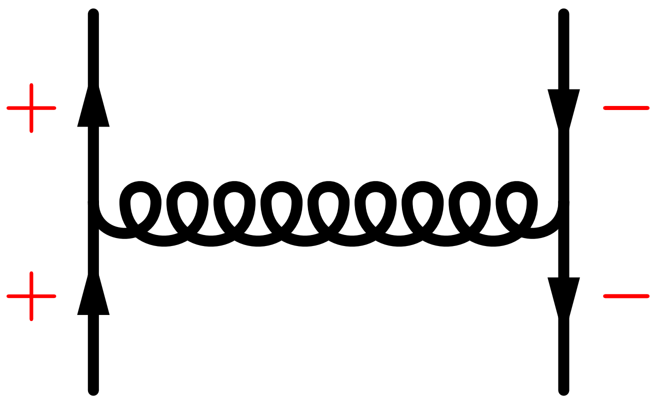
Transversity is one of three twist-two structures:

$$
\begin{aligned}
f(x) &= \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle \\
\Delta f(x) &= \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle \\
\Delta_T f(x) &= \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle PS|\bar{\psi}(0)\gamma^+\gamma^1\gamma_5\psi(0,\xi^-,\mathbf{0}_\perp)|PS\rangle
\end{aligned}
$$

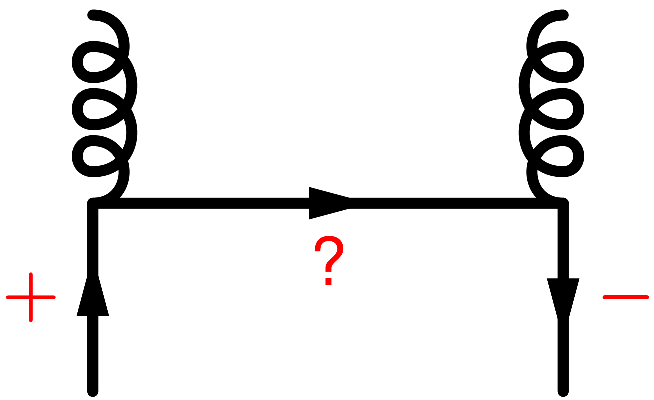
The γ_5 matrix signals spin dependence.

The extra γ^1 matrix in $\Delta_T f(x)$ signals the helicity-flip that precludes transversity contributions in DIS.

Ladder Diagram Summation



universal evolution kernel
in a physical (axial) gauge
for transversity



gluon–fermion mixing
not allowed

LO QCD evolution of transversity is non-singlet like

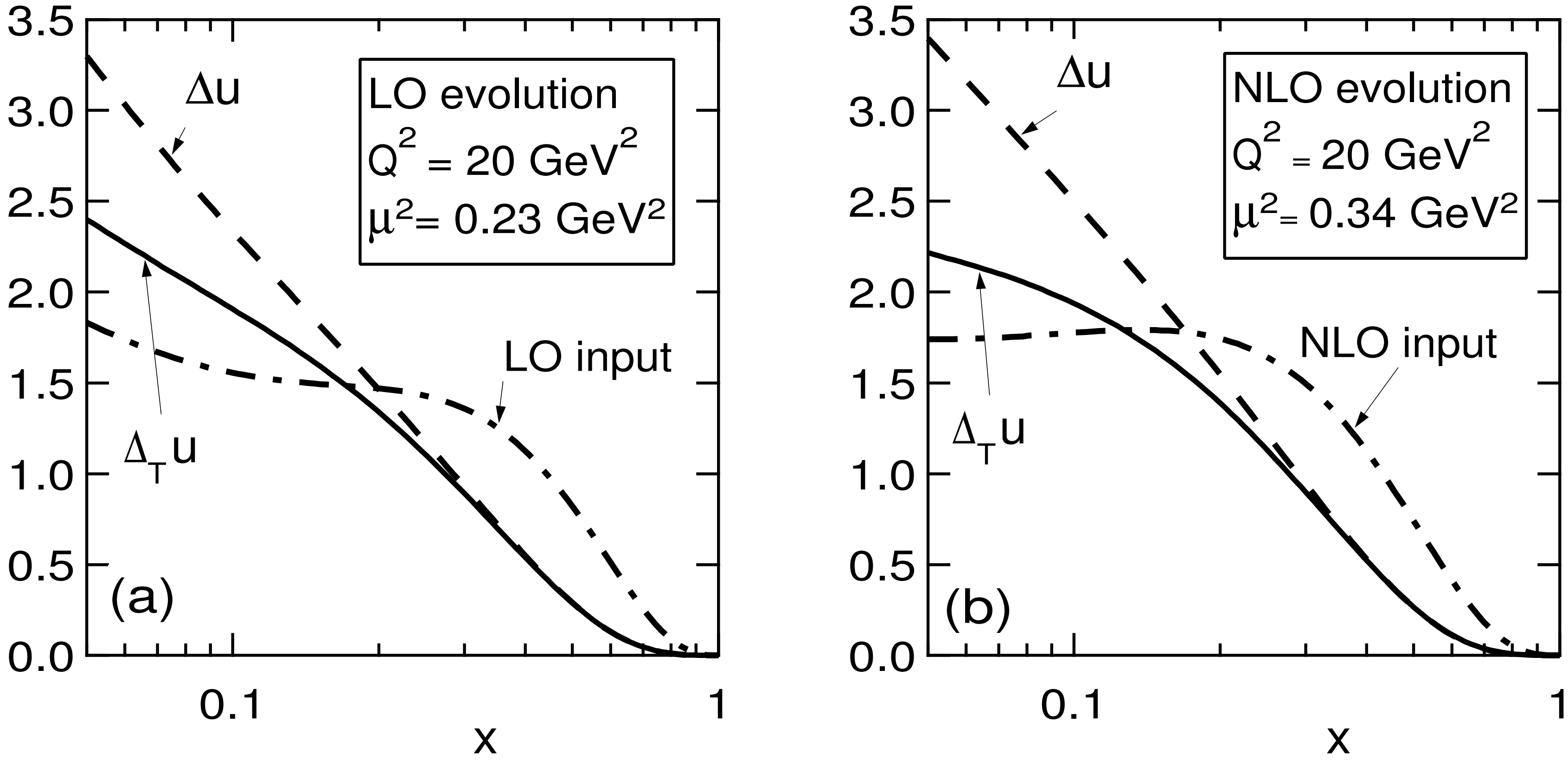
Leading Order DGLAP

The LO DGLAP quark–quark splitting functions:

$$
\begin{aligned}
P_{qq}^{(0)} &= C_{\mathsf{F}} \left(\frac{1+x^2}{1-x} \right)_+ \\
\Delta P_{qq}^{(0)} &= P_{qq}^{(0)} \quad \text{helicity conservation} \\
\Delta_T P_{qq}^{(0)} &= C_{\mathsf{F}} \left[\left(\frac{1+x^2}{1-x} \right)_+ - 1 + x \right] \\
&= P_{qq}^{(0)}(x) - C_{\mathsf{F}}(1-x)
\end{aligned}
$$

N.B. For both $P_{qq}^{(0)}$ and $\Delta P_{qq}^{(0)}$ the first moments vanish (leading to conservation laws and sum rules) . . . but not for $\Delta_T P_{qq}^{(0)}$

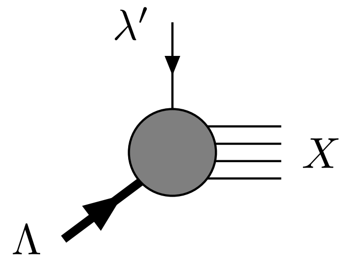
LO and NLO Evolution



Comparison of the Q^2-evolution of $\Delta_T u(x, Q^2)$ and $\Delta u(x, Q^2)$ at (a) LO and (b) NLO

Soffer Bound

Soffer (1995)

hadron–parton amplitudes: $a_{\Lambda,\lambda'} \sim$ 

$$\begin{array}{rclcl} f(x) & \propto & \mathrm{Im}(\mathcal{A}_{++,++} + \mathcal{A}_{+-,+-}) & \propto & \sum_X (a^*_{++} a_{++} + a^*_{+-} a_{+-}) \\ \Delta f(x) & \propto & \mathrm{Im}(\mathcal{A}_{++,++} - \mathcal{A}_{+-,+-}) & \propto & \sum_X (a^*_{++} a_{++} - a^*_{+-} a_{+-}) \\ \Delta_T f(x) & \propto & \mathrm{Im}\,\mathcal{A}_{+-,-+} & \propto & \sum_X a^*_{--} a_{++} \end{array}$$

$$\sum_X |a_{++} \pm a_{--}|^2 \geq 0 \quad \Rightarrow \quad \sum_X a^*_{++} a_{++} \pm \sum_X a^*_{--} a_{++} \geq 0$$

leads to

$f_+(x) \geq |\Delta_T f(x)|$ or $\boxed{f(x) + \Delta f(x) \geq 2|\Delta_T f(x)|}$

A DIS Definition for Transversity

The other twist-2 functions are naturally defined in DIS, where the parton model is usually formulated and model calculations performed.

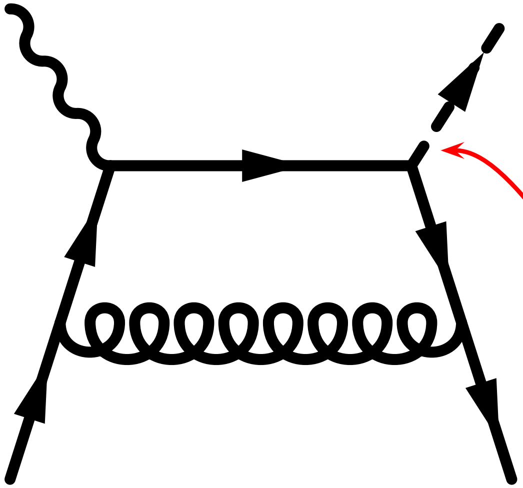
When translated to DY, large K factors appear $\sim$ O$(\pi\alpha_s)$.

At RHIC energies this is a 30% correction, at EMC/SMC energies it is nearly 100%.

Pure DY coefficient functions are known, but are scheme dependent. Moreover, a $\frac{\ln^2 x}{1-x}$ term appears that is not found for spin-averaged or helicity-dependent DY.

Added to the problems arising with the Vector–Scalar current product this suggests an interesting check . . .

DIS Higgs–Photon Interference

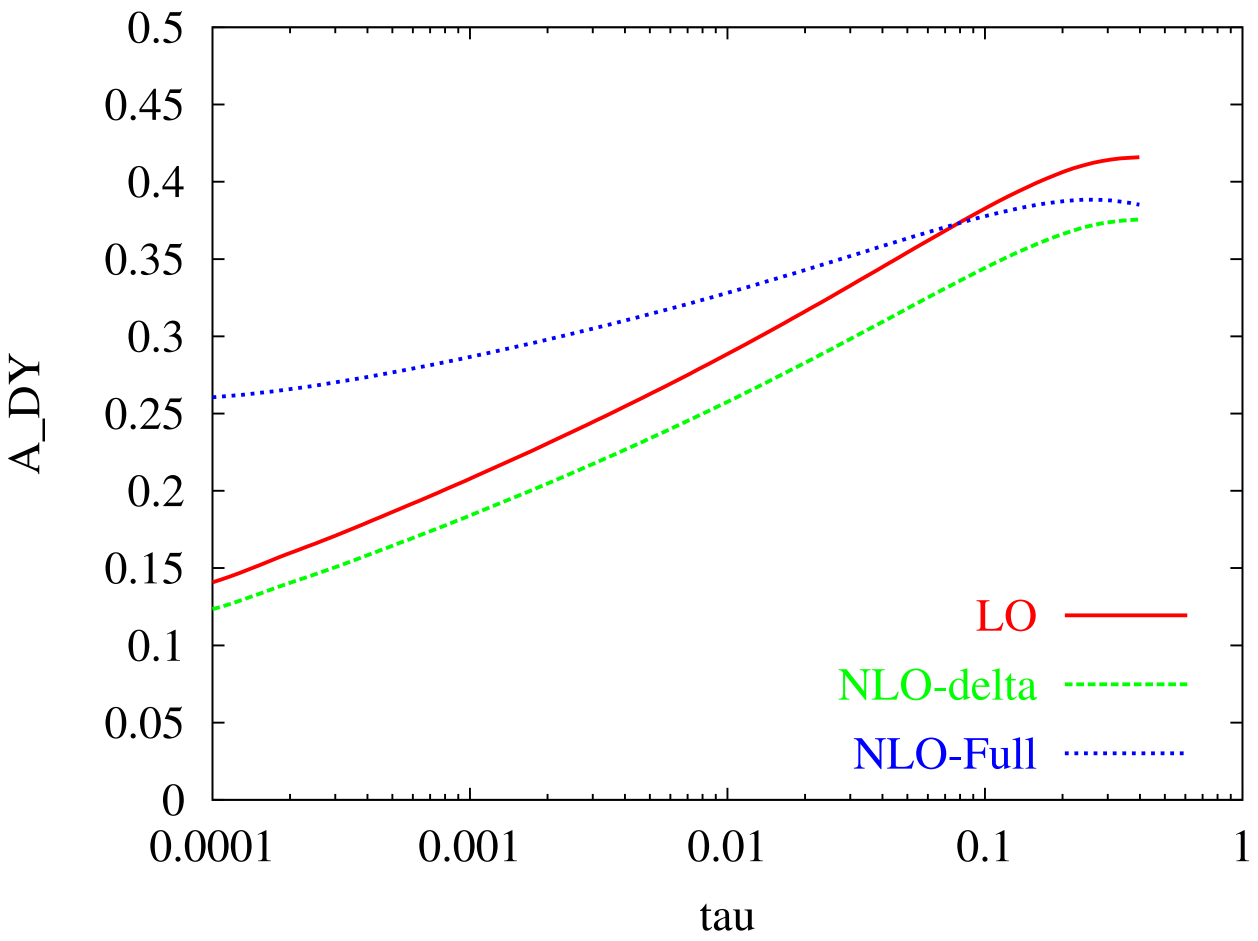


The extra contribution from the scalar vertex is factorised into the running mass (or Higgs coupling constant).

$$C^f_{q,DY} - 2C^f_{q,DIS} = \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\frac{3}{(1-z)_+} + 2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 6 - 4z + \left(\frac{4}{3}\pi^2 + 1\right)\delta(1-z)\right]$$

$$C^h_{q,DY} - 2C^h_{q,DIS} = \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\frac{3z}{(1-z)_+} + 4z\left(\frac{\ln(1-z)}{1-z}\right)_+ - 6z\frac{\ln^2 z}{1-z} + 4(1-z) + \left(\frac{4}{3}\pi^2 - 1\right)\delta(1-z)\right]$$

DIS–DY Asymmetry



Transversity asymmetry (valence only) for Drell–Yan.
[$\tau = Q^2/s$, $s = 4{\cdot}10^4$ GeV^2, kinematic limits $\tau < x1, x2 < 1$]

Notation

$\boldsymbol{k}_\perp$-integrated distribution functions:

$f(x)$	number density,
$\Delta f(x)$	helicity distributions,
$\Delta_T f(x)$	transverse-polarisation distributions,

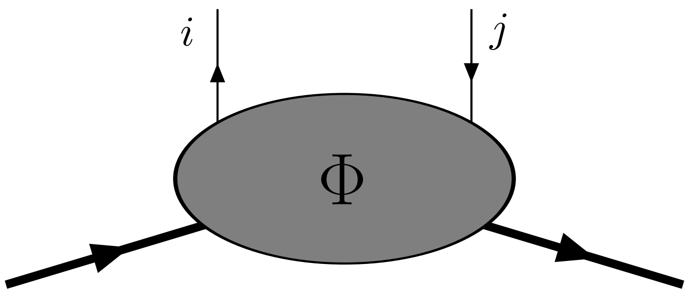
Objects like $\Delta_L^T f$ have a fairly simple interpretation:

- *subscripts* $0, L$ and T in distribution and fragmentation functions denote the *quark* polarisation state
- *superscripts* $0, L$ and T denote the parent or off-spring *hadron* polarisation state

The superscript is dropped when equal to the subscript.

Correlation Matrix

The aim is parametrise the quark–quark correlation matrix:



respecting the properties of hermiticity, parity, and time-reversal invariance.
The most general decomposition of Φ in a basis of Dirac matrices is

$$\Phi(k, P, S) = \frac{1}{2}\left\{\mathcal{S}\,\mathbb{1} + \mathcal{V}_\mu\,\gamma^\mu + \mathcal{A}_\mu\gamma_5\gamma^\mu + \mathrm{i}\mathcal{P}_5\gamma_5 + \mathrm{i}\,\mathcal{T}_{\mu\nu}\,\sigma^{\mu\nu}\gamma_5\right\}.$$

The quantities $\mathcal{S}$, $\mathcal{V}^\mu$, $\mathcal{A}^\mu$, $\mathcal{P}_5$ and $\mathcal{T}^{\mu\nu}$ are to be constructed with the vectors k^μ, P^μ and the pseudovector S^μ.

T-Odd Structures

Relaxing T invariance allows two new twist-two structures:

$$\mathcal{V}^{\mu} = \cdots + \frac{1}{M} A_1' \, \varepsilon^{\mu\nu\rho\sigma} \, P_{\nu} k_{\perp\rho} S_{\perp\sigma}$$

$$\mathcal{T}^{\mu\nu} = \cdots + \frac{1}{M} A_2' \, \varepsilon^{\mu\nu\rho\sigma} \, P_{\rho} k_{\perp\sigma}$$

which give rise to two $\boldsymbol{k}_{\perp}$-dependent T-odd distribution functions, $f_{1T}^{\perp}$ and $h_1^{\perp}$ (Boer and Mulders, 1998)

$$\Phi^{[\gamma^+]} = \cdots - \frac{\varepsilon_{\perp}^{ij} k_{\perp i} S_{\perp j}}{M} \, f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp}^2)$$

$$\Phi^{[\mathrm{i}\sigma^{i+}\gamma_5]} = \cdots - \frac{\varepsilon_{\perp}^{ij} k_{\perp j}}{M} \, h_1^{\perp}(x, \boldsymbol{k}_{\perp}^2)$$

Partonic Interpretation

The first of the new distributions, $f_{1T}^{\perp}$, is related to the number density of unpolarised quarks in a transversely polarised nucleon:

$$\begin{aligned}\mathcal{P}_{q/N\uparrow}(x, \boldsymbol{k}_\perp) - \mathcal{P}_{q/N\downarrow}(x, \boldsymbol{k}_\perp) \\ &= \mathcal{P}_{q/N\uparrow}(x, \boldsymbol{k}_\perp) - \mathcal{P}_{q/N\uparrow}(x, -\boldsymbol{k}_\perp) \\ &= -2 \frac{|\boldsymbol{k}_\perp|}{M} \sin(\phi_k - \phi_S)\, f_{1T}^{\perp}(x, \boldsymbol{k}_\perp^2)\end{aligned}$$

The other T-odd distribution, $h_1^{\perp}$, measures quark transverse polarisation in an unpolarised hadron:

$$\mathcal{P}_{q\uparrow/N}(x, \boldsymbol{k}_\perp) - \mathcal{P}_{q\downarrow/N}(x, \boldsymbol{k}_\perp) = -\frac{|\boldsymbol{k}_\perp|}{M} \sin(\phi_k - \phi_s)\, h_1^{\perp}(x, \boldsymbol{k}_\perp^2)$$

Partonic Interpretation

It is convenient to define two quantities $\Delta_0^T f$ and $\Delta_T^0 f$, related respectively to $f_{1T}^{\perp}$ and $h_1^{\perp}$, by absorbing the explicit factors $|\boldsymbol{k}_\perp|/M$:

$$\begin{aligned} \Delta_0^T f(x,\boldsymbol{k}_\perp^2) &\equiv -2\frac{|\boldsymbol{k}_\perp|}{M} f_{1T}^{\perp}(x,\boldsymbol{k}_\perp^2) \\ \Delta_T^0 f(x,\boldsymbol{k}_\perp^2) &\equiv -\frac{|\boldsymbol{k}_\perp|}{M} h_1^{\perp}(x,\boldsymbol{k}_\perp^2) \end{aligned}$$

Now the question arises as to why we should be willing to entertain such T-odd quantities . . .

T-Odd Justification

There are various approaches:

Anselmino and Murgia (1998) (among others) advocate initial-state effects, which prevent implementation of naïve time-reversal invariance.

The idea, similar to that which leads to admitting T-odd fragmentation functions as a result of final-state effects, is that the colliding particles interact strongly with non-trivial relative phases.

T-Odd Justification

An alternative way of viewing T-odd distributions has been proposed by Anselmino *et al*. (2002).

Applying a general argument on time reversal for particle multiplets suggested by Weinberg (1995), the argument is that, if the internal structure of hadrons is described at some low momentum scale by a chiral lagrangian, time reversal might be realised in a “non-standard” manner that could mix the multiplet components.

In fact, with this idea, the u (d) distribution transforms into the d (u) distribution, and time-reversal invariance simply establishes a relation between the u and d sectors.

T-Odd Justification

Finally very recently, Collins (2002) has reconsidered his proof of the vanishing of $f_{1T}^{\perp}$ and $h_1^{\perp}$, based on the field-theoretical expressions of the two distributions.

He has noticed that, if one reinstates the link operators into quark–quark bilocals, the distributions do not simply change sign under T, because a future-pointing Wilson line is transformed into a past-pointing Wilson line.

Consequently, time-reversal invariance, does not constrain $f_{1T}^{\perp}$ and $h_1^{\perp}$ to zero, but relates processes probing Wilson lines in opposite directions. So, Collins predicts the Sivers asymmetry to have opposite signs in DIS and in DY.

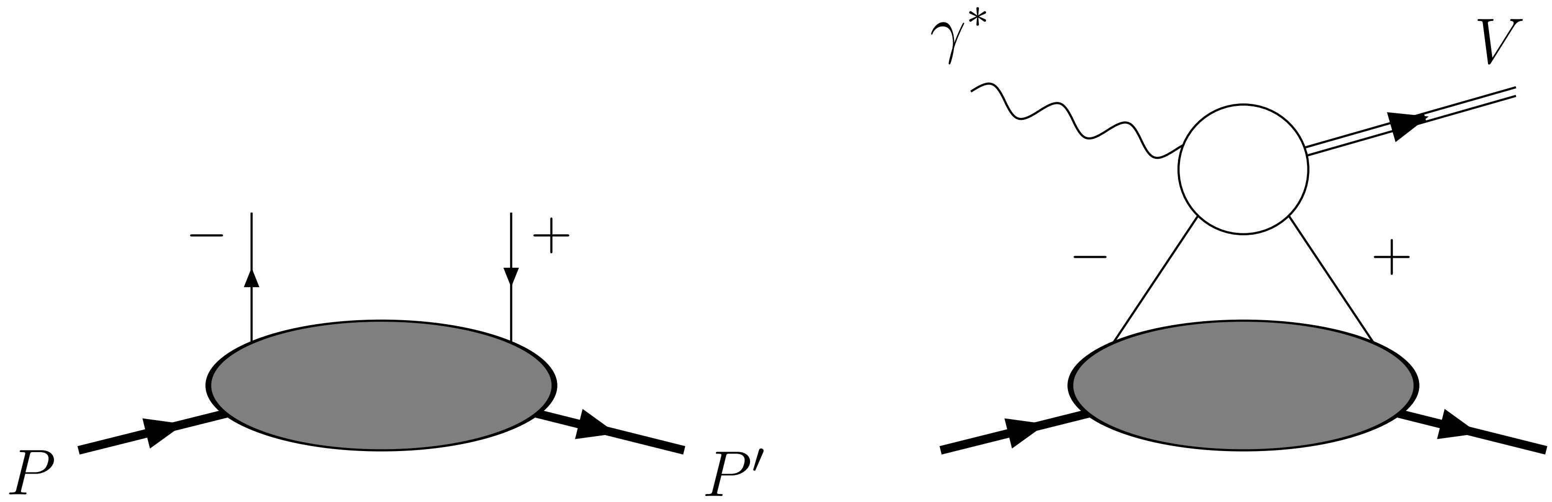
Lepton-Nucleon Scattering

Topics:

- Exclusive Processes
- Single Longitudinal-Spin Asymmetries

Exclusive Processes

One might hope to access transversity through exclusive leptoproduction of vector mesons:



Mankiewicz, Piller and Weigl (1998) showed that the chirally-odd contribution to vector-meson production is actually zero at LO in α_s.

Exclusive Processes

Diehl, Gousset and Pire (1999) and Collins and Diehl (2000) later extended this, observing that the chirally-odd contribution vanishes due to angular momentum and chirality conservation in the hard scattering and so holds at leading twist to all orders in α_s.

Thus, the (off-diagonal) transversity distributions cannot be probed in exclusive vector-meson leptoproduction.

Single Longitudinal-Spin

The cross-section for a longitudinally polarised target was given by Kotzinian and Mulders (1997):

$$\frac{\mathrm{d}^5\sigma(\lambda_N)}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P}_{h\perp}} = -\frac{4\pi\alpha_{\text{em}}^2 s}{Q^4}\,\lambda_N \sum_a e_a^2\, x(1-y)\,\sin(2\phi_h)$$

$$\times\, I\left[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_\perp)(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp) - \boldsymbol{\kappa}_\perp\cdot\boldsymbol{k}_\perp}{M\,M_h}\, h_{1La}^\perp(x,\boldsymbol{k}_\perp)\, H_{1a}^\perp(z,\boldsymbol{\kappa}_\perp)\right]$$

No transversity, but depends on the Collins function $H_1^\perp$, $\propto \sin(2\phi_h)$, also a $\boldsymbol{k}_\perp$-dependent distribution function $h_{1L}^\perp$. The x and z dependence can be factorised by weighting the cross-sections with functions of azimuthal angles.

Leptoproduction Summary

Summarising the situation in the context of semi-inclusive DIS there are four candidate reactions for determining $\Delta_T f$ at leading twist:

1. inclusive leptoproduction of a transversely polarised hadron from a transversely polarised target;
2. inclusive leptoproduction of an unpolarised hadron from a transversely polarised target;
3. inclusive leptoproduction of two hadrons from a transversely polarised target;
4. inclusive leptoproduction of a spin-one polarised or unpolarised hadron from a transversely polarised target.

Hadron-Hadron Scattering

Topics:

- Single-Particle Production
  - Transverse-Momentum Effects
  - Twist-Three Effects
- Drell–Yan

Single-Hadron Production

Single-hadron production with a transversely polarised target:

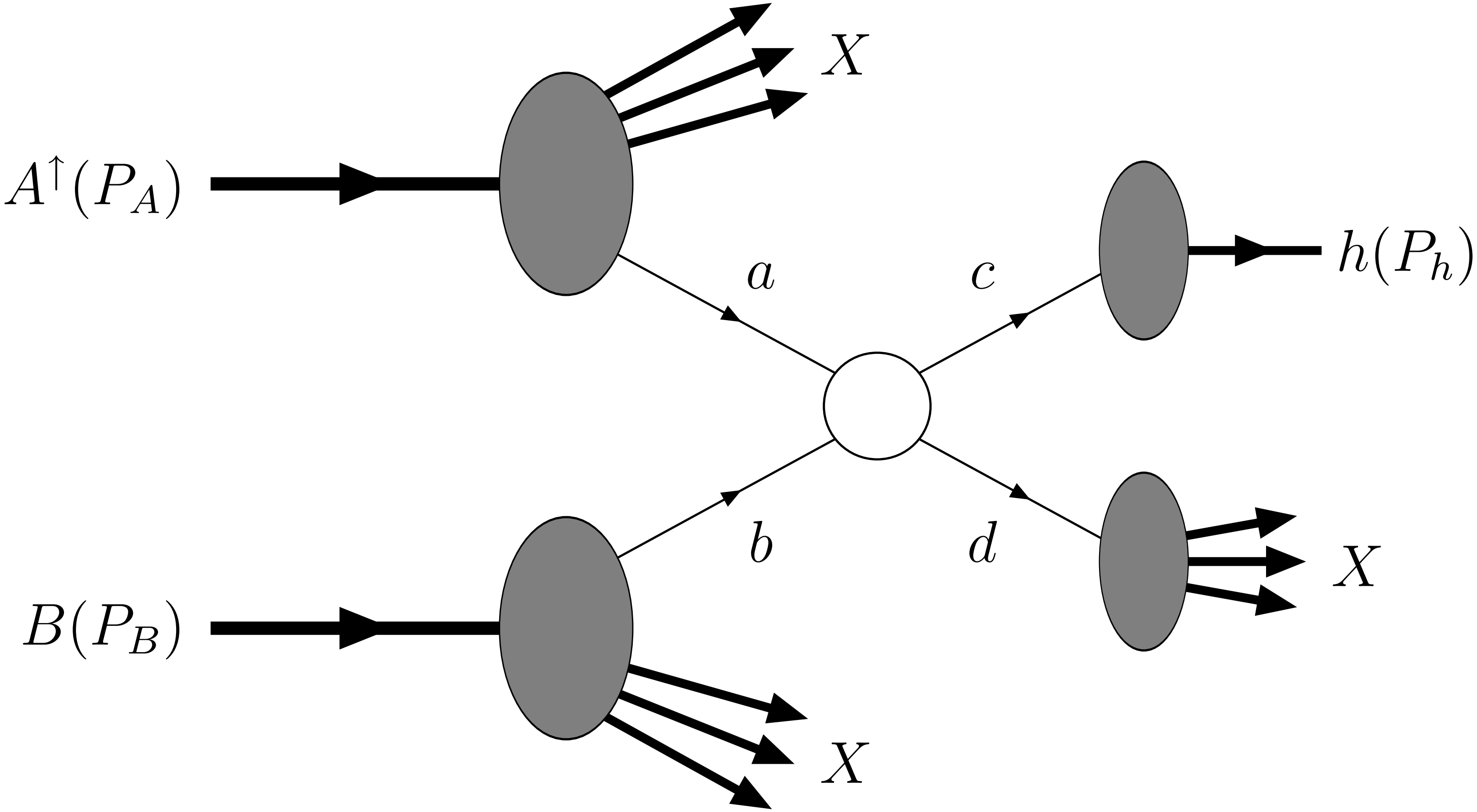
$$A^{\uparrow}(P_A) + B(P_B) \rightarrow h(P_h) + X$$

A is transversely polarised and the unpolarised (or spinless) hadron h is produced at large transverse momentum $\boldsymbol{P}_{hT}$, thus PQCD is applicable.

In typical experiments A and B are protons while h is a pion. One measures an SSA:

$$A_T^h = \frac{\mathrm{d}\sigma(\boldsymbol{S}_T) - \mathrm{d}\sigma(-\boldsymbol{S}_T)}{\mathrm{d}\sigma(\boldsymbol{S}_T) + \mathrm{d}\sigma(-\boldsymbol{S}_T)}$$

Single-Hadron Production



Single-Hadron Production

According to the factorisation theorem, the differential cross-section for the reaction may be written formally as

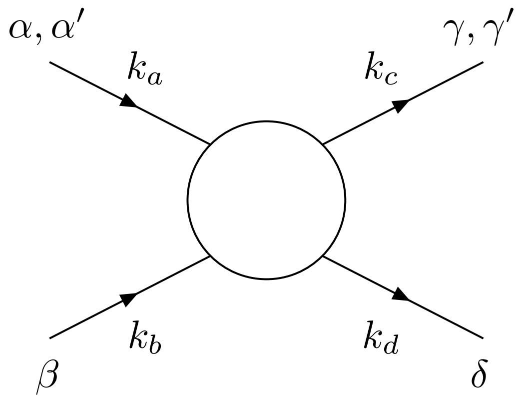
$$\mathrm{d}\sigma = \sum_{abc} \sum_{\alpha\alpha'\gamma\gamma'} \rho^a_{\alpha'\alpha}\, f_a(x_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma}_{\alpha\alpha'\gamma\gamma'} \otimes \mathcal{D}^{\gamma'\gamma}_{h/c}(z)$$

Here f_a (f_b) is the density of parton a (b) inside hadron A (B), $\rho^a_{\alpha\alpha'}$ is the spin density matrix of parton a, $\mathcal{D}^{\gamma\gamma'}_{h/c}$ is the fragmentation matrix of parton c into hadron h and $\mathrm{d}\hat{\sigma}/\mathrm{d}\hat{t}$ is the elementary cross-section:

$$\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}\right)_{\alpha\alpha'\gamma\gamma'} = \frac{1}{16\pi\hat{s}^2}\,\frac{1}{2} \sum_{\beta\delta} \mathcal{M}_{\alpha\beta\gamma\delta}\, \mathcal{M}^*_{\alpha'\beta\gamma'\delta}$$

Single-Hadron Production

$\mathcal{M}_{\alpha\beta\gamma\delta}$ is the hard partonic scattering amplitude:



For an unpolarised produced hadron, the off-diagonal elements of $\mathcal{D}_{h/c}^{\gamma\gamma'}$ vanish, *i.e.*, $\mathcal{D}_{h/c}^{\gamma\gamma'} \propto \delta_{\gamma\gamma'}$. Then helicity conservation implies $\alpha = \alpha'$ and there is no dependence on the spin of hadron A, so all SSA's are zero.

To avoid this conclusion, either intrinsic quark transverse motion, or higher-twist effects must be considered . . .

Single-Hadron Production

Quark intrinsic transverse motion can generate SSA's in three different ways:

1. $\boldsymbol{\kappa}_T$ in hadron h implies $\mathcal{D}_{h/c}^{\gamma\gamma'}$ may be non-diagonal (T-odd effect at the fragmentation level).
2. $\boldsymbol{k}_T$ in hadron A implies that $f_a(x_a)$ should be replaced by the $\mathcal{P}_a(x_a, \boldsymbol{k}_T)$, which may depend on the spin of hadron A (T-odd effect at the distribution level).
3. $\boldsymbol{k}'_T$ in hadron B implies that $f_b(x_b)$ in should be replaced by $\mathcal{P}_b(x_b, \boldsymbol{k}'_T)$. The transverse spin of parton b in the unpolarised hadron B may then couple to the transverse spin of parton a in A (T-odd effect at the distribution level).

Transverse Motion and SSA

1. is the Collins effect (1993)
2. is the Sivers effect (1990)
3. is an effect studied by Boer (1999) in the context of DY processes

It should be stressed that all these intrinsic-$\boldsymbol{\kappa}_T$, -$\boldsymbol{k}_T$, or -$\boldsymbol{k}'_T$ effects are T-odd.

Note too that when intrinsic quark transverse motion is taken into account, the QCD factorisation theorem is not proven.

Transverse Motion and SSA

Assume factorisation is valid the cross-section is

$$E_h \frac{\mathrm{d}^3\sigma}{\mathrm{d}^3\boldsymbol{P}_h} = \sum_{abc} \sum_{\alpha\alpha'\beta\beta'\gamma\gamma'} \int \mathrm{d}x_a \int \mathrm{d}x_b \int \mathrm{d}^2\boldsymbol{k}_T \int \mathrm{d}^2\boldsymbol{k}'_T \int \mathrm{d}^2\boldsymbol{\kappa}_T \, \frac{1}{\pi z}$$

$$\times \, \mathcal{P}_a(x_a, \boldsymbol{k}_T) \, \rho^a_{\alpha'\alpha} \, \mathcal{P}_b(x_b, \boldsymbol{k}'_T) \, \rho^b_{\beta'\beta}$$

$$\times \left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \right)_{\alpha\alpha'\beta\beta'\gamma\gamma'} \mathcal{D}^{\gamma'\gamma}_{h/c}(z, \boldsymbol{\kappa}_T)$$

where

$$\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} \right)_{\alpha\alpha'\beta\beta'\gamma\gamma'} = \frac{1}{16\pi\hat{s}^2} \sum_{\beta\delta} \mathcal{M}_{\alpha\beta\gamma\delta} \mathcal{M}^*_{\alpha'\beta\gamma'\delta}$$

Transverse Motion and SSA

The Collins mechanism requires we take into account the intrinsic quark transverse motion inside the produced hadron h, and neglect the transverse momenta of all other quarks (assuming the spin of A to be directed along y):

$$E_h \frac{\mathrm{d}^3\sigma(\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} - E_h \frac{\mathrm{d}^3\sigma(-\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h}$$

$$= -2\,|\boldsymbol{S}_T| \sum_{abc} \int \mathrm{d}x_a \int \frac{\mathrm{d}x_b}{\pi z} \int \mathrm{d}^2\boldsymbol{\kappa}_T$$

$$\times\, \Delta_T f_a(x_a)\, f_b(x_b)\, \Delta_{TT}\hat{\sigma}(x_a, x_b, \boldsymbol{\kappa}_T)\, \Delta_T^0 D_{h/c}(z, \boldsymbol{\kappa}_T^2)$$

$\Delta_{TT}\hat{\sigma}$ is a partonic spin-transfer asymmetry.

Transverse Motion and SSA

The Sivers effect relies on T-odd distribution functions and predicts a single-spin asymmetry of the form

$$E_h \frac{\mathrm{d}^3\sigma(\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} - E_h \frac{\mathrm{d}^3\sigma(-\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h}$$

$$= |\boldsymbol{S}_T| \sum_{abc} \int \mathrm{d}x_a \int \frac{\mathrm{d}x_b}{\pi z} \int \mathrm{d}^2\boldsymbol{k}_T$$

$$\times \Delta_0^T f_a(x_a, \boldsymbol{k}_T^2)\, f_b(x_b)\, \frac{\mathrm{d}\hat{\sigma}(x_a, x_b, \boldsymbol{k}_T)}{\mathrm{d}\hat{t}}\, D_{h/c}(z)$$

where $\Delta_0^T f$ (related to $f_{1T}^{\perp}$) is a T-odd distribution.

Transverse Motion and SSA

Finally, the effect studied by Boer (1999) gives rise to an asymmetry involving the other T-odd distribution, $\Delta_T^0 f$ (related to $h_1^\perp$):

$$E_h \, \frac{\mathrm{d}^3\sigma(\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h} - E_h \, \frac{\mathrm{d}^3\sigma(-\boldsymbol{S}_T)}{\mathrm{d}^3\boldsymbol{P}_h}$$

$$= -2|\boldsymbol{S}_T| \sum_{abc} \int \mathrm{d}x_a \int \frac{\mathrm{d}x_b}{\pi z} \int \mathrm{d}^2\boldsymbol{k}'_T$$

$$\times \, \Delta_T f_a(x_a) \, \Delta_T^0 f_b(x_b, \boldsymbol{k}'^2_T) \, \Delta_{TT}\hat{\sigma}'(x_a, x_b, \boldsymbol{k}'_T) \, D_{h/c}(z)$$

$\Delta_{TT}\hat{\sigma}'$ is a partonic initial-spin correlation asymmetry.

Transverse Motion and SSA

Efremov and Teryaev (1982) first pointed out that non-vanishing SSA's can also be obtained in PQCD by resorting to higher twist and the so-called gluonic poles present in diagrams involving qqg correlators.

Such asymmetries were later evaluated in the context of QCD factorisation by Qiu and Sterman, who studied direct photon production (1991; 1992) and, more recently, hadron production (1999).

This program has been extended to cover the chirally-odd contributions by Kanazawa and Koike (2000a,b).

Transverse Motion and SSA

$$
\begin{aligned}
\mathrm{d}\sigma \;=\; \sum_{abc} \Big\{ & G_F^a(x_a, y_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma} \otimes D_{h/c}(z) \\
& + \Delta_T f_a(x_a) \otimes E_F^b(x_b, y_b) \otimes \mathrm{d}\hat{\sigma}' \otimes D_{h/c}(z) \\
& + \Delta_T f_a(x_a) \otimes f_b(x_b) \otimes \mathrm{d}\hat{\sigma}'' \otimes D_{h/c}^{(3)}(z) \Big\}
\end{aligned}
$$

The first term does not contain the transversity distribution and is a chirally-even mechanism studied by Qiu and Sterman.

The second term is the chirally-odd contribution analysed by Kanazawa and Koike.

The third contains a twist-three fragmentation function $D_{h/c}^{(3)}$.

Drell–Yan at Twist Three

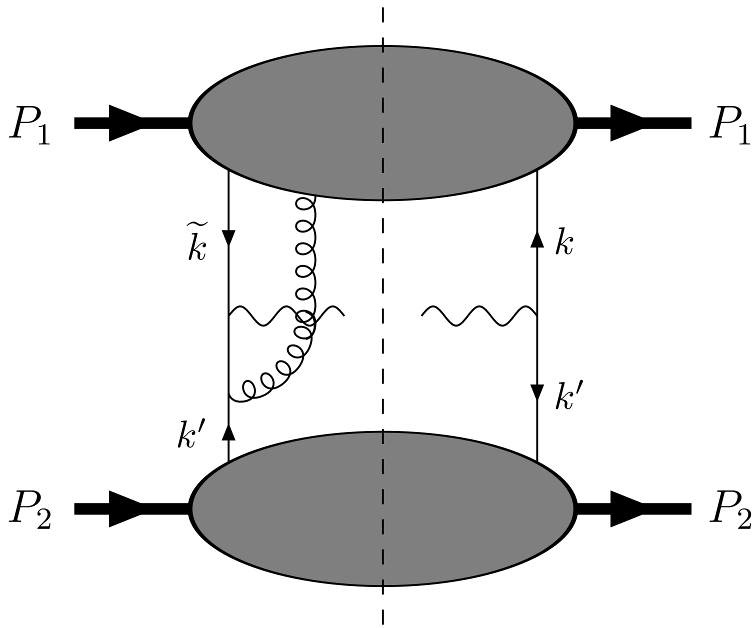
Admitting twist-three contributions, the single-spin asymmetry in DY is—Boer, Mulders and Teryaev (1997)

$$A_T^{\mathrm{DY}} = |\boldsymbol{S}_{1\perp}| \frac{2\sin 2\theta}{1+\cos^2\theta} \sin(\phi - \phi_{S_1}) \frac{M}{Q} \times \frac{\sum_a e_a^2 \left[x_1 \tilde{f}_T^a(x_1)\, \bar{f}_a(x_2) + x_2\, \Delta_T f_a(x_1)\, \bar{h}_a(x_2)\right]}{\sum_a e_a^2\, f_a(x_1) \bar{f}_a(x_2)} .$$

$\widetilde{f}_T(x)$ and $\bar{h}(x)$ are twist-three T-odd distribution functions
The existence of T-odd distribution functions has been advocated by Boer (1999) to explain an anomalously large $\cos 2\phi$ term in unpolarised DY.
As given, requires initial-state interactions—unlikely?

Drell–Yan at Twist Three

Hammon, Teryaev and Schäfer (1997) have shown that single-spin asymmetries may arise from gluonic poles in twist-three multiparton correlation functions:



Drell–Yan at Twist Three

The single-spin asymmetry is then

$$A_T^{\mathsf{DY}} \quad \propto \quad |\boldsymbol{S}_{1\perp}| \, \frac{2\sin 2\theta}{1+\cos^2\theta} \, \sin(\phi - \phi_{S_1}) \, \frac{M}{Q}$$
$$\times \, \frac{\sum_a e_a^2 \left[G_F^a(x_1, x_1) \, \bar{f}_a(x_2) + \Delta_T f_a(x_1) \, E_F^a(x_2, x_2) \right]}{\sum_a e_a^2 \, f_a(x_1) \bar{f}_a(x_2)}$$

Comparing with the previous expression we may identify

$$\begin{aligned} f_T^{\mathsf{eff}}(x) \quad &\sim \quad G_F(x,x) \quad \sim \quad \int \mathrm{d}y \; \mathsf{Im}\, G_A^{\mathsf{eff}}(x,y) \\ h^{\mathsf{eff}}(x) \quad &\sim \quad E_F(x,x) \quad \sim \quad \int \mathrm{d}y \; \mathsf{Im}\, E_A^{\mathsf{eff}}(x,y) \end{aligned}$$

Thus, T-odd functions at twist three, can explain A_T^{DY} via quark–gluon interactions, without initial-state effects.

Conclusions

The study of single-spin asymmetries has become a rather complex and almost involuted area of high-energy spin physics.

The plethora of new structure functions and fragmentation functions alike opens up the possibility of explaining many of the old processes that have begged a theory for many years.

However, in order to separate out all these effects and distinguish between the various possibilities a large amount of diverse high-energy data will be necessary and it is hard to point a finger at a few key experiments.

References

Andersson B., Gustafson G. and Ingelman G. (1979), *Phys. Lett.* **B85**, 417.

Anselmino M. and Murgia F. (1998), *Phys. Lett.* **B442**, 470.

Anselmino M., Barone V., Drago A. and Murgia F. (2002), paper in preparation.

Antoniadis I. and Kounnas C. (1981), *Phys. Rev.* **D24**, 505.

Artru X. and Mekhfi M. (1990), *Z. Phys.* **C45**, 669.

Baldracchini F., Craigie N.S., Roberto V. and Socolovsky M. (1981), *Fortschr. Phys.* **30**, 505.

Barni R., Preparata G. and PGR (1992), *Phys. Lett.* **B296**, 251.

Barone V., Drago A. and PGR (2002), *Phys. Rep.* **359**, 1.

Barone V. and PGR (in preparation), *Transverse Polarisation of Quarks in Hadrons* (World Sci.).

Boer D. (1999), *Phys. Rev.* **D60**, 014012.

Boer D., Mulders P.J. and Teryaev O.V. (1997), *Phys. Rev.* **D57**, 3057.

Boer D. and Mulders P.J. (1998), *Phys. Rev.* **D57**, 5780.

Bukhvostov A.P., Kuraev É.A., Lipatov L.N. and Frolov G.V. (1985), *Nucl. Phys.* **B258**, 601.

Collins J.C. (1993), *Nucl. Phys.* **B396**, 161.

Collins J.C. (2002), *Phys. Lett.* **B536**, 43.

Collins J.C. and Diehl M. (2000), *Phys. Rev.* **D61**, 114015.

DeGrand T.A. and Miettinen H.I. (1981), *Phys. Rev.* **D24**, 2419; *erratum*, *ibid*. **D31** (1985) 661.

Diehl M., Gousset T. and Pire B. (1999), *Phys. Rev.* **D59**, 034023.

Efremov A.V. and Teryaev O.V. (1982), *Yad. Fiz.* **36**, 242; *transl.*, *Sov. J. Nucl. Phys.* **36** (1982) 140.

Efremov A.V. and Teryaev O.V. (1985), *Phys. Lett.* **B150**, 383.

Hammon N., Teryaev O. and Schäfer A. (1997), *Phys. Lett.* **B390**, 409.

Kanazawa Y. and Koike Y. (2000a), *Phys. Lett.* **B478**, 121.

Kanazawa Y. and Koike Y. (2000b), *Phys. Lett.* **B490**, 99.

Kane G.L., Pumplin J. and Repko W. (1978), *Phys. Rev. Lett.* **41**, 1689.

Kodaira J., Matsuda S., Sasaki K. and Uematsu T. (1979), *Nucl. Phys.* **B159**, 99.

Kotzinian A.M. and Mulders P.J. (1997), *Phys. Lett.* **B406**, 373.

Mankiewicz L., Piller G. and Weigl T. (1998), *Eur. Phys. J.* **C5**, 119.

PGR (1986), *Nucl. Phys.* **B264**, 493.

Qiu J.-W. and Sterman G. (1991), *Phys. Rev. Lett.* **67**, 2264.

Qiu J.-W. and Sterman G. (1992), *Nucl. Phys.* **B378**, 52.

Qiu J.-W. and Sterman G. (1999), *Phys. Rev.* **D59**, 014004.

Ralston J. and Soper D.E. (1979), *Nucl. Phys.* **B152**, 109.

Sivers D. (1990), *Phys. Rev.* **D41**, 83.

Soffer J. (1995), *Phys. Rev. Lett.* **74**, 1292.

Soffer J. and Tornqvist N.A. (1992), *Phys. Rev. Lett.* **68**, 907.

Weinberg S. (1995), *The Quantum Theory of Fields. Vol. 1: Foundations* (Cambridge U. Press).